Topic 6

Superposition Theorem
Thevenin's Theorem
Norton's Theorem

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Superposition Theorem

The current through or voltage across an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

Let's use this circuit (fig. 4.1) to illustrate the method:

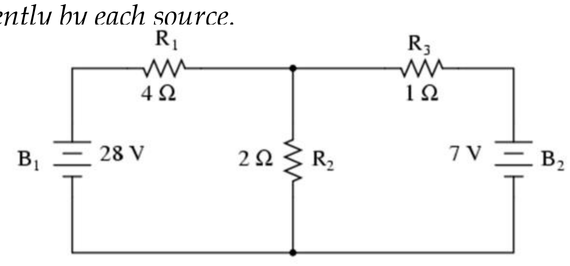


Figure: 4.1

Calculation:

Considering the effect of 28-V source (fig. 4.2);

$$R_T = R_1 + R_2 \parallel R_3 = 4 + 2 \parallel 1 = 4.67\Omega$$

$$I_1' = \frac{B_1}{R_T} = \frac{28}{4.67} = 6A$$

$$I_2' = \frac{I_1' \times R_3}{R_2 + R_3} = \frac{6 \times 1}{3} = 2A$$

$$I_3' = \frac{I_1' \times R_2}{R_2 + R_3} = \frac{6 \times 2}{3} = 4A$$

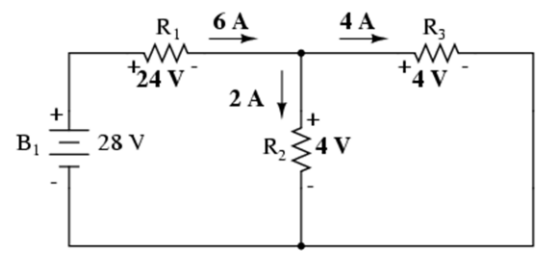


Figure: 4.2

Considering the effect of 7-V source (fig. 4.3);

$$R_T = R_3 + R_1 \parallel R_2 = 1 + 4 \parallel 2 = 2.33\Omega$$

$$I_3'' = \frac{B_2}{R_T} = \frac{7}{2.33} = 3A$$

$$I_2'' = \frac{I_3'' \times R_1}{R_1 + R_2} = \frac{3 \times 4}{6} = 2A$$

$$I_1'' = \frac{I_3'' \times R_2}{R_1 + R_2} = \frac{3 \times 2}{6} = 1A$$

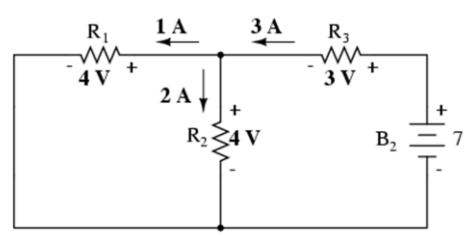


Figure: 4.3

Now considering the current directions we get (fig. 4.4)

$$I_1 = I_1' - I_1'' = 6 - 1 = 5A$$

$$I_2 = I_2' + I_2'' = 2 + 2 = 4A$$

$$I_3 = I_3' - I_3'' = 4 - 3 = 1A$$

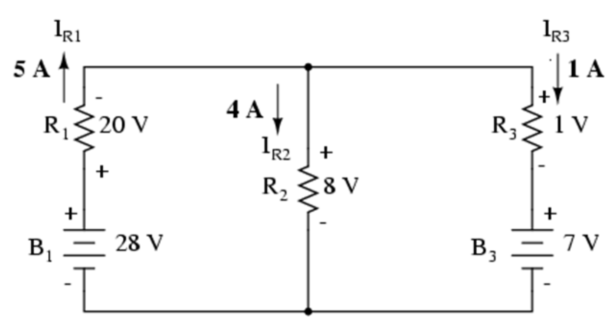


Figure: 4.4

Thevenin's Theorem

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

Steps to follow for Thevenin's Theorem:

(1) Find the Thevenin resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculate the total resistance between the open connection points.

(2) Find the Thevenin source voltage by removing the load resistor from the original circuit and calculate the voltage across the open connection points where the load resistor used to be.

(3) Draw the Thevenin equivalent circuit, with the Thevenin voltage source in series with the Thevenin resistance. The load resistor re-attaches between the two open points of the equivalent circuit.

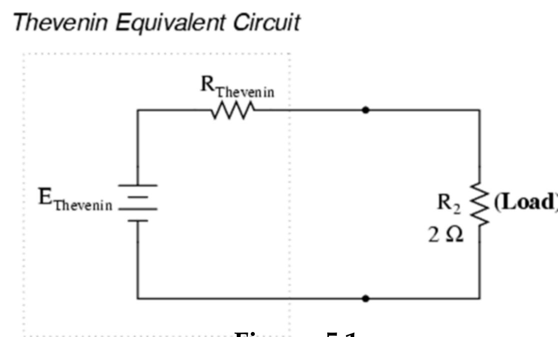


Figure: 5.1

Let's use this circuit (fig. 5.2) to illustrate the method:

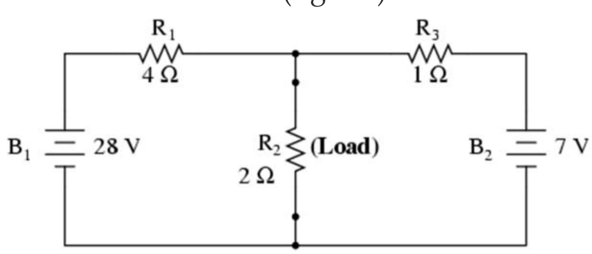


Figure: 5.2

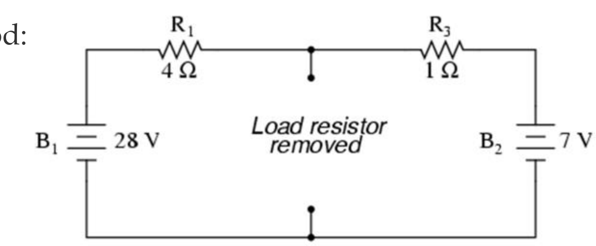


Figure: 5.3

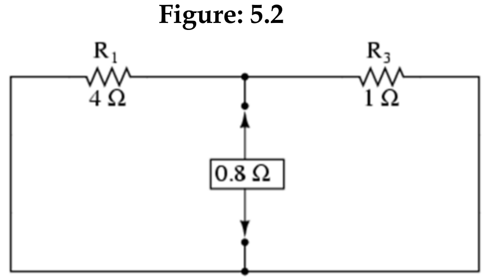


Figure: 5.4

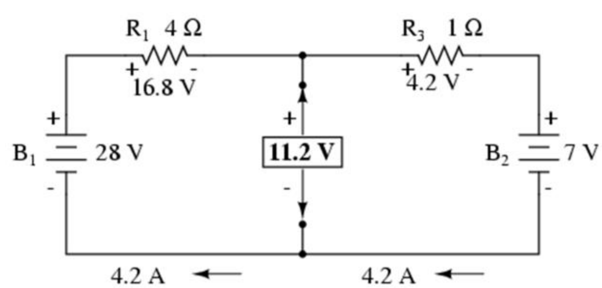


Figure: 5.4

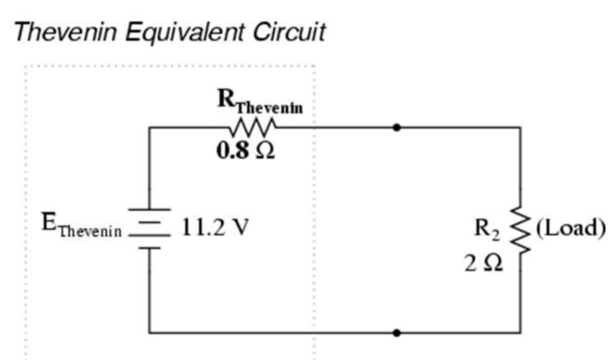


Figure: 5.5

Norton's Theorem

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.

Steps to follow for Norton's Theorem:

(1) Find the Norton resistance by removing all power sources in the original circuit (voltage sources shorted and current sources open) and calculating total resistance between the open connection points.

(2) Find the Norton source current by removing the load resistor from the original circuit and calculating current through a short (wire) jumping across the open connection points where the load resistor used to be.

(3) Draw the Norton equivalent circuit, with the Norton current source in parallel with the Norton resistance. The load resistor re-attaches between the two open points of the equivalent circuit.

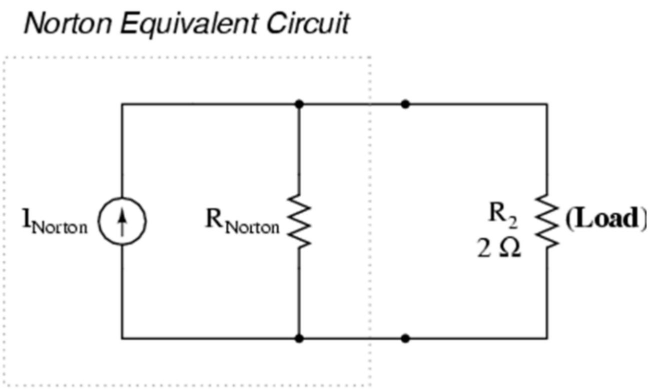


Figure: 6.1

Let's use this circuit (fig. 6.2) to illustrate the method:

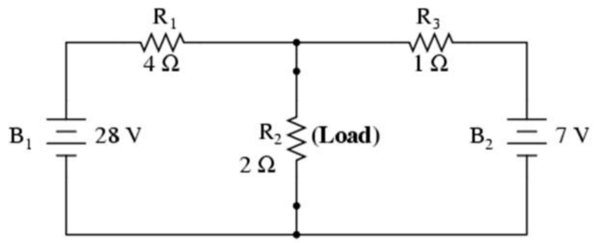


Figure: 6.2

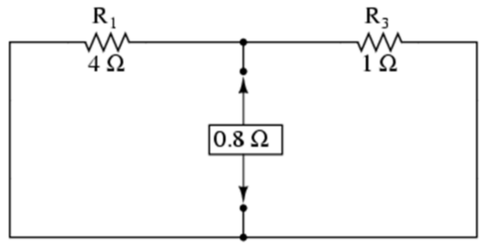


Figure: 6.3

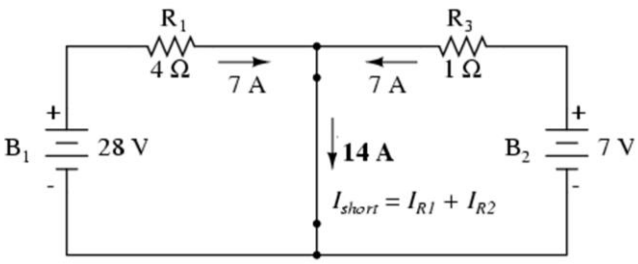


Figure: 6.4

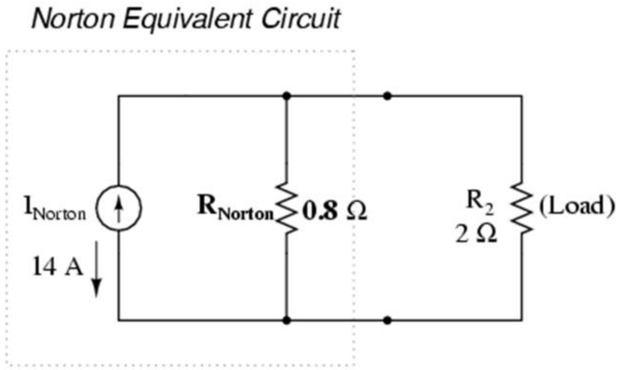


Figure: 6.5

Sample Answer script_Superposition-Thevenin-Norton

Question 1: Determine the currents through and voltages across each branch of the network shown in figure Q1 using superposition theorem.

Question 2: Draw the Thevenin's equivalent circuit across the marked terminal of the network shown in figure Q2 and determine the currents through the marked load.

Question 3: Draw the Norton's equivalent circuit across the marked terminal of the network shown in figure Q2. Show that the current through the marked load determined using the superposition theorem, Norton and Thevenin are same.

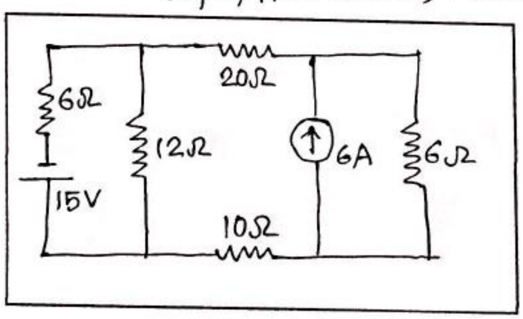


Figure (without marked load) Q1

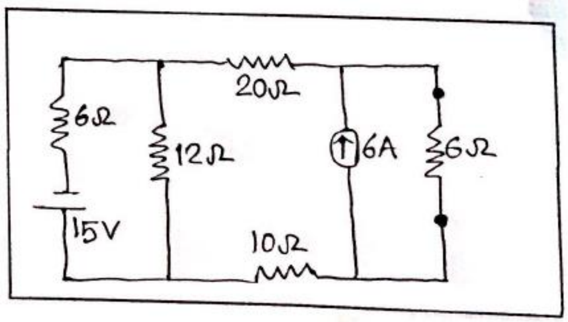
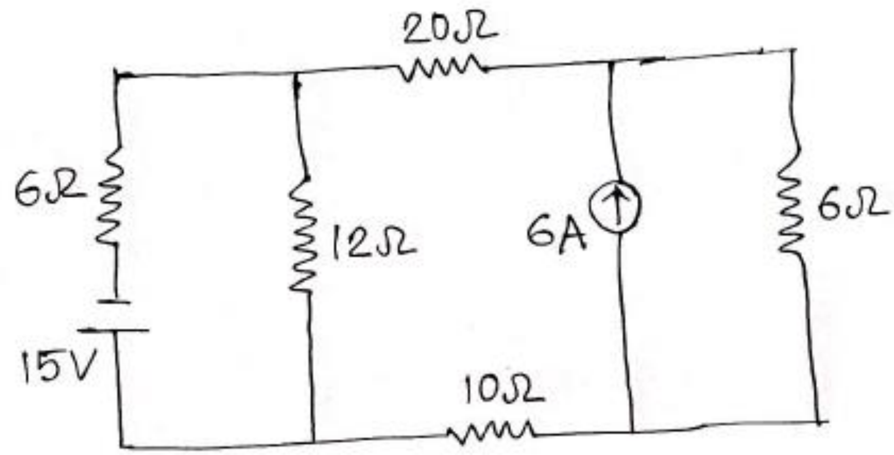
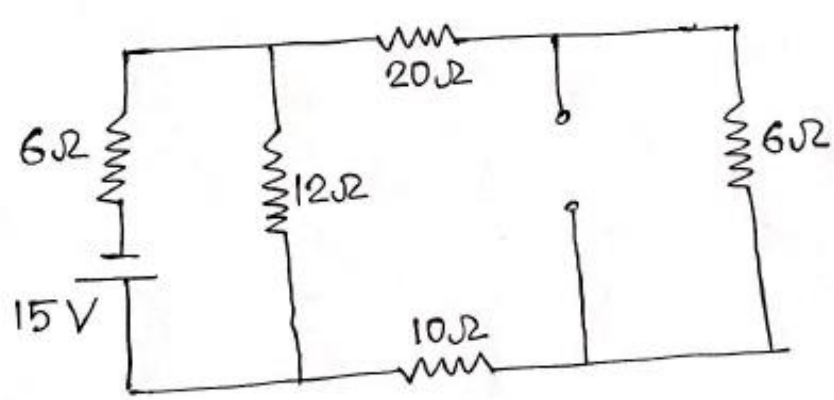


Figure (with marked load) Q2

Ans. to the Question no- (1)



Considering the effect of 15 V source,



$$I'_{6\Omega(left)} = \frac{15}{((20+6+10)||12)+6} = \frac{15}{15} = 1A \downarrow$$

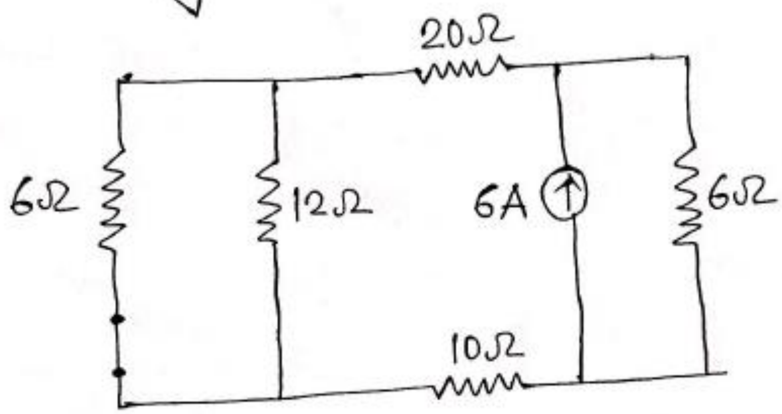
$$I'_{12\Omega} = \frac{1 \times (20+6+10)}{12+(20+6+10)} = 0.75 A \uparrow$$

$$I'_{10\Omega} = 1-0.75 = 0.25 A \rightarrow$$

$$I'_{6\Omega(right)} = 0.25 A \uparrow$$ (in series with 10Ω)

$$I'_{20\Omega} = 0.25 A \leftarrow$$

Cosidering the effect of 6A current source,



The whole left part to 6A is in parallel to the 6Ω resistor at right of it.

$$\therefore I''_{6\Omega(\text{right})} = \frac{6 \times \{(6\|12)+20+10\}}{((6\|12)+20+10)+6} = 5.1A\downarrow$$

$$I''_{20\Omega} = (6-5.1) = 0.9A \leftarrow$$

$$I''_{10\Omega} = 0.9A \longrightarrow$$ (in series with 20Ω and the equivalent of 6Ω and 12Ω)

$$I''_{6\Omega(\text{left})} = \frac{0.9 \times 12}{12+6} = 0.6A\downarrow$$

$$I''_{12\Omega} = 0.9 - 0.6 = 0.3A\downarrow$$

Now, considering the current directions we get,

$$I_{6\Omega(left)} = I'_{6\Omega(left)} + I''_{6\Omega(left)} = 1+0.6 = 1.6A \downarrow$$

$$I_{12\Omega} = I'_{12\Omega} - I''_{12\Omega} = 0.75-0.3 = 0.45A \uparrow$$

$$I_{10\Omega} = I'_{10\Omega} + I''_{10\Omega} = 0.25+0.9 = 1.15A \rightarrow$$

$$I_{6\Omega(right)} = I''_{6\Omega(right)} - I'_{6\Omega(right)} = 5.1-0.25 = 4.85 \downarrow$$

$$I_{20\Omega} = I'_{20\Omega} + I''_{20\Omega} = 0.25+0.9 = 1.15A \leftarrow$$

The voltages across each of the resistors,

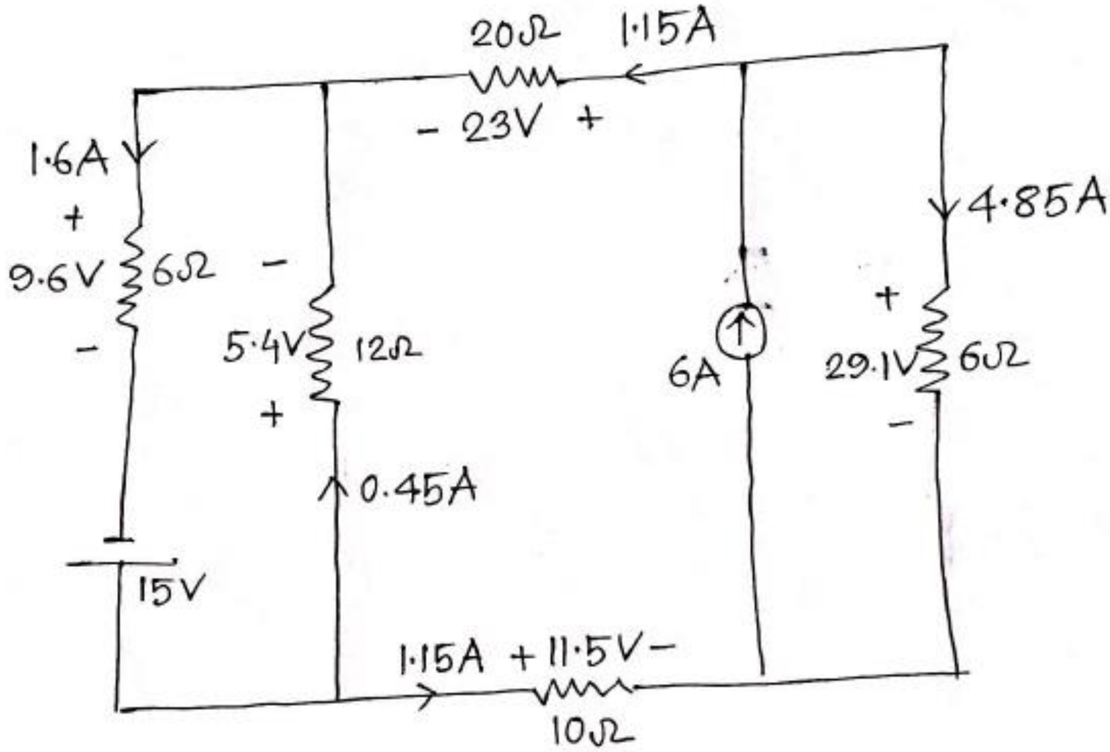
$$V_{6\Omega(left)} = 6\times1.6 = 9.6V$$

$$V_{12\Omega} = 12\times0.45 = 5.4V$$

$$V_{10\Omega} = 10\times1.15 = 11.5V$$

$$V_{6\Omega(right)} = 6\times4.85 = 29.1V$$

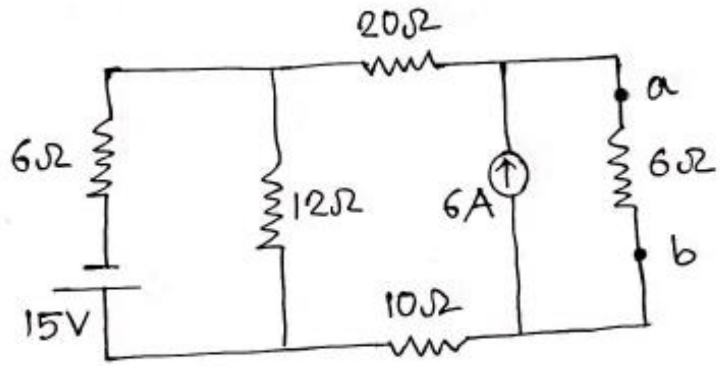
$$V_{20\Omega} = 20\times1.15 = 23V$$



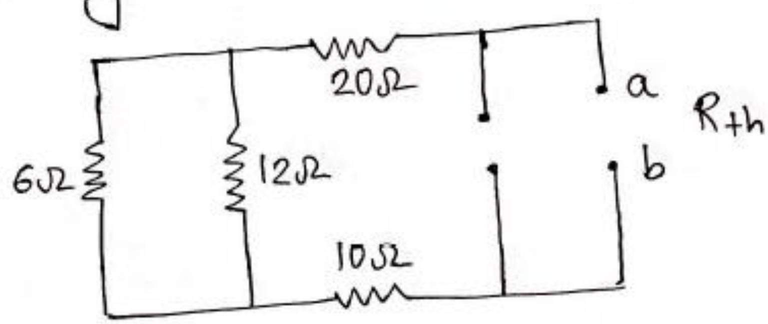
Here, the current through the 6Ω resistor, (at the right)

$I_{superposition} = 4.85A \downarrow$

Ans. to the Question no -(2)

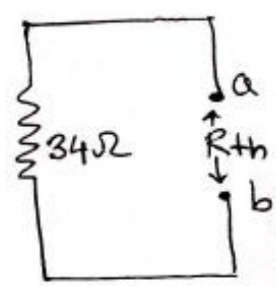


Removing the marked load resistor, and turning all sources inactive,



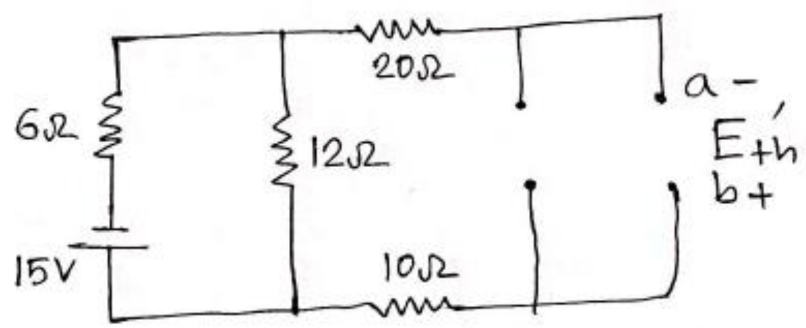
∴ Thevenin equivalent resistance,

$$R_{th} = (6 \parallel 12) + 10 + 20$$
$$= 4 + 30$$
$$= 34\Omega$$



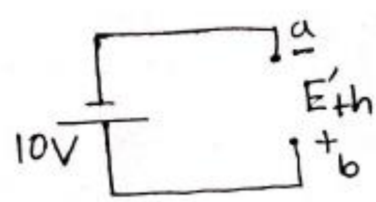
Now, we shall apply superposition theorem to find the thevenin voltage across the marked load.

Cosidering the 15V ~~load~~ source as active,

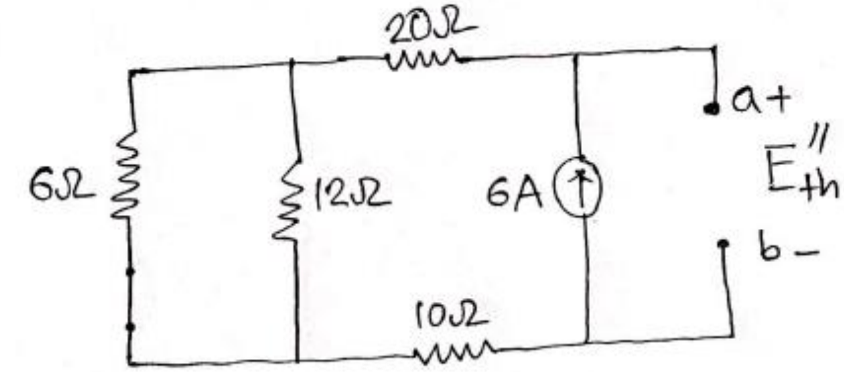


As there is no current through the 10Ω and 20Ω resistors, the voltage across 12Ω will be the voltage at the a−b.

$$\therefore E'_{th} = \frac{12 \times 15}{12+6} = 10\,V$$



Now, considering the 6A current source as active,

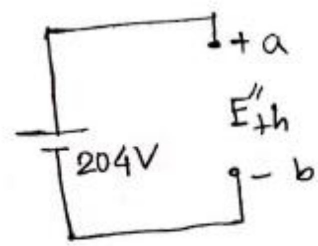


Here, $R_T = (6 \| 12) + 20 + 10$

$= 34\Omega$

$\therefore E_{th}'' = (34 \times 6)$

$= 204\text{ V}$

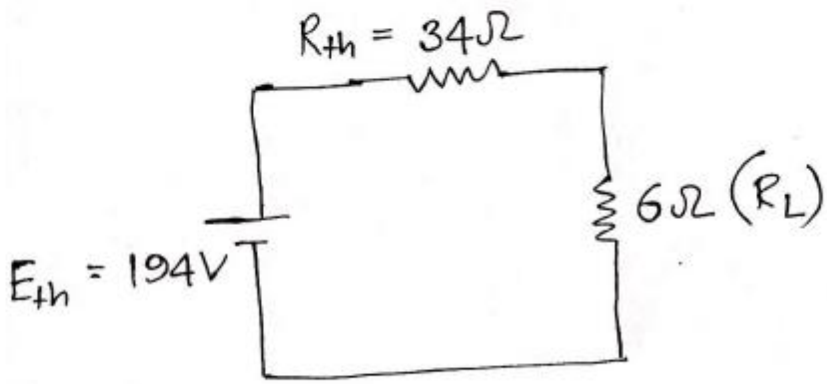


$\therefore E_{th} = E_{th}'' - E_{th}'$

$= 204\text{ V} - 10\text{ V}$

$= 194\text{ V}$

Therefore, the thevenin equivalent circuit,

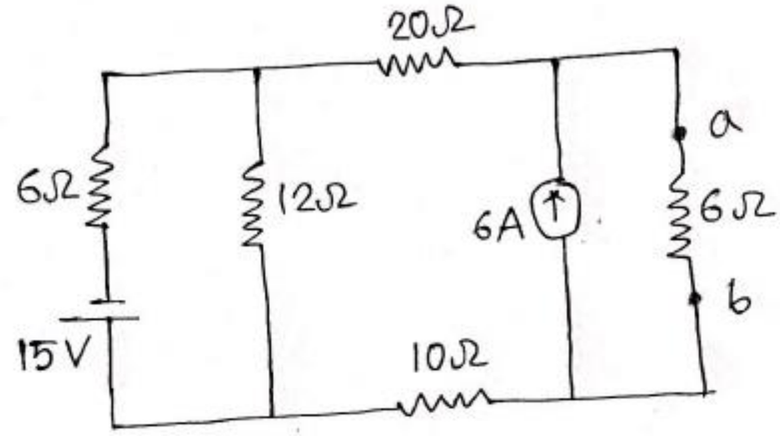


The current through the marked load,

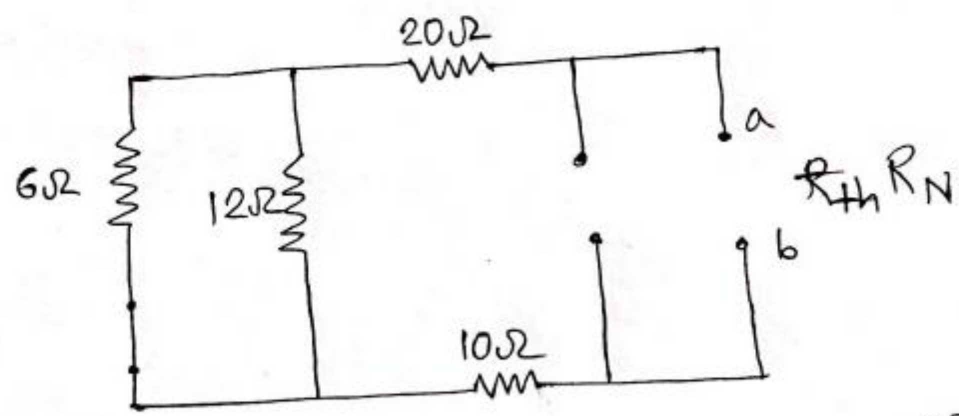
$$I_{thevenin} = \frac{194V}{(34+6)\Omega}$$

$$\therefore I_{thevenin} = 4.85A \downarrow$$

Ans. to the Question no-(3)

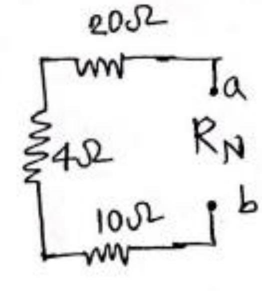


Removing the marked load and making all the sources inactive,



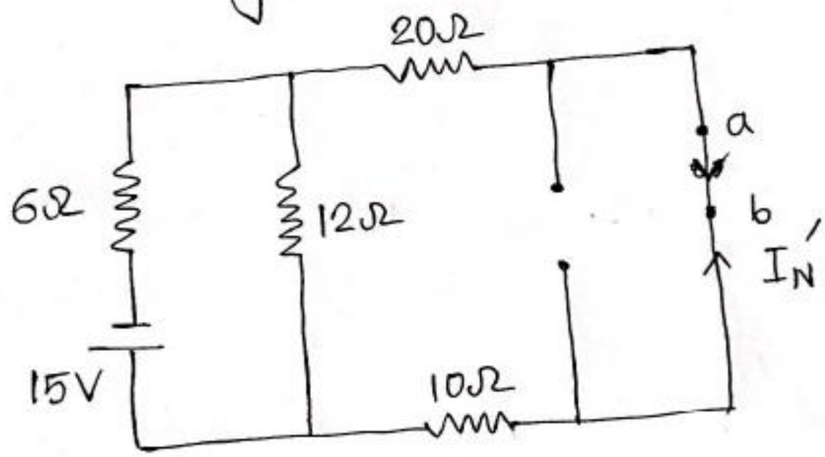
Norton Equivalent Resistance,

$$R_N = (6 \parallel 12) + 10 + 20$$
$$= 34\Omega$$



Now, connecting the open marked load with a short line we apply the superposition theorem to find the Norton source current.

Considering the 15V as active,



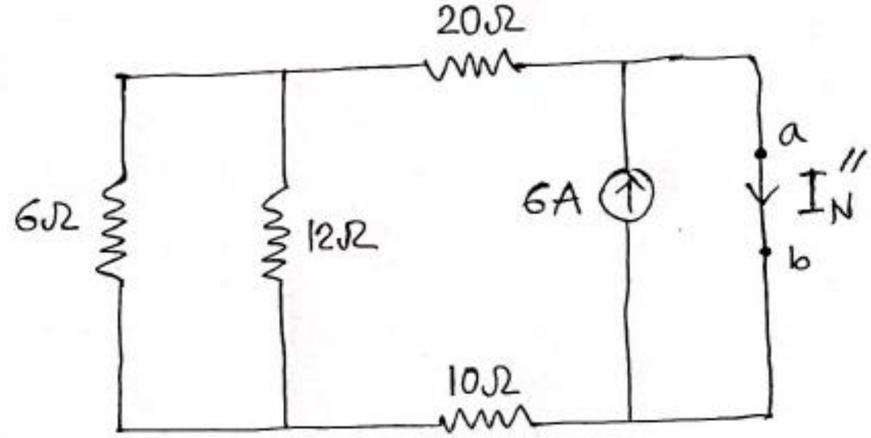
Here, $R_{Total} = ((20+10) \| 12) + 6$

$= 14.57\,\Omega$

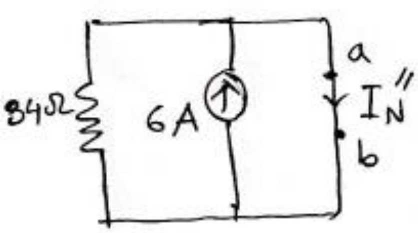
$$\therefore I_N' = \frac{\frac{15}{14.57} \times 12}{(20+10)+12}$$

$= 0.29\ A \downarrow$

Considering the 6A current source as active,



Since there is a short line parallell to the resistors, the complete 6A current will flow through it.



$\therefore I_N'' = 6A \uparrow$

$\therefore I_N = 6A - 0.29A$

$= 5.71A \uparrow$

Therefore, the Norton equivalent circuit for across the marked terminal,

Now, the current through the marked load,

$$I_{Norton} = \frac{5.71 \times 34}{34+6}$$

$$= 4.85A \downarrow$$

We can see from the previous problem,

$$I_{Thevenin} = 4.85A \downarrow$$

and $I_{superposition} = 4.85A \downarrow$

Therefore, $I_{superposition} = I_{Thevenin} = I_{Norton}$

Self-study

Q. 1) Using superposition theorem determine the current through the resistor (indicating) of the figure shown below.

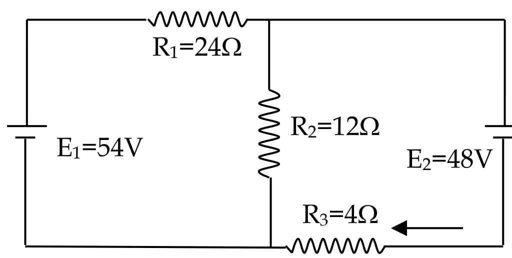


Figure: 1(a)

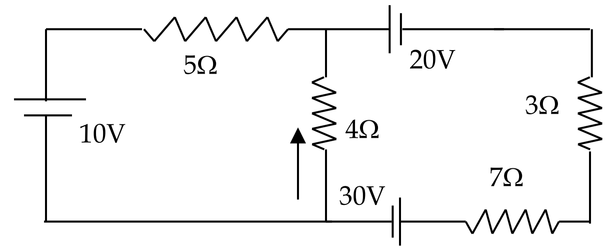


Figure: 1(b)

Q. 2) Determine the Norton's equivalent and Thevenin's equivalent circuit across the resistor (indicating) for the network of the figure given below.

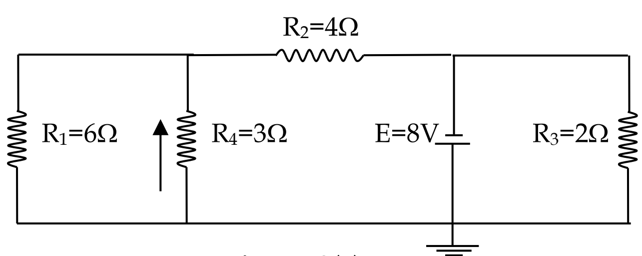


Figure: 2(a)

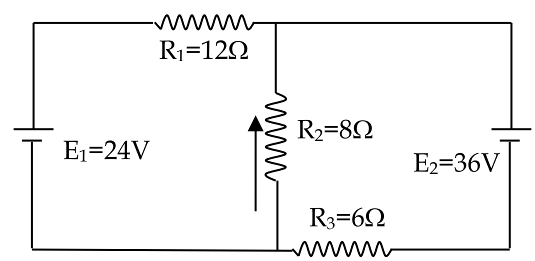


Figure: 2(b)

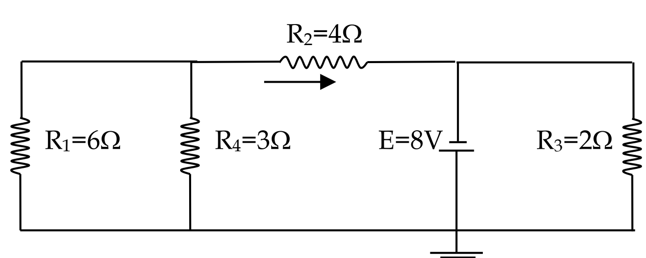


Figure: 2(c)